Study of the Mach-Zehnder Interferometric Technique for Dielectric Resonator Tuning

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ABSTRACT: The modulation of a microwave-scale Mach-Zehnder interferometer coupled to a dielectric resonator (DR) tunes the DR resonance frequency. We report on a theoretical study to develop an understanding of the intrinsic tuning properties of a Mach-Zehnder interferometer ring-DR-microstrip line system and present an experimental verification.

Keywords: dielectric resonator; tuning; satellite communications; Mach-Zehnder interferometer

I. INTRODUCTION

Rings and ring-based components, such as, resonators and filters, have been extensively studied [1–4]. Little work has been done, however, on applying rings as agents to effect the tuning of dielectric resonators (DR) [5]. In this novel context, it is conceptually profitable to think of the ring as operating as a Mach-Zehnder interferometer (MZI) and to explore how, when coupled to a DR, modulation of one of the MZI paths causes tuning of the DR. This approach is believed to be important because it paves the way to high-Q-preserving DR tuning. This article presents an extended version of a conference article [6], where a theoretical study, and experimental verification, of the intrinsic tuning properties of a metallic microwave-scale MZI-DR-transmission line system, of the type employed in, e.g., DR oscillators, was first presented. Section II presents an overview of the MZI ring and motivates its utilization as a tuning agent for DRs. Section III introduces the MZI-DR-microstrip line system, develops an understanding of its operation, a circuit model, and presents the experimental verification of DR tuning using this scheme.

II. MZI-BASED TUNING CONCEPT

A. Overview of the MZI

The MZI, in its implementation as a metallic microstrip ring, is depicted in Figure 1. It consists of two parallel curved paths, i = 1, 2, forming an annular ring, characterized by propagation constant, \( k_i = \omega \sqrt{L_i/C_i} \), where \( \omega \) is the wave frequency and \( L_i \) and \( C_i \), the inductance and capacitance per unit length, respectively. When the propagation constants of the two ring arms are equal, the input wave, after being split at the input junction, propagates down both paths and is recreated at the output junction, where recombination occurs. On the other hand, causing the propagation constants of the two ring paths to be unequal results in wave amplitude at the output that exhibits a degree of interference. Assuming unit input wave amplitude and equal splitting in both ring arms, the output wave amplitude is

\[
T = \left| t \right|^2 = 1 + \cos \left( L \cdot (k_2-k_1) \right),
\]

where \( L \) is half the ring mean circumference. Clearly, the maximum of \( T \) is obtained whenever the cosine argument equals zero or an integer multiple of \( 2\pi \). Now, if the MZI ring had no outgoing transmission line, it is easy to see that there could be no transmission, and, therefore, the waves on each ring arm would counter propagate, establishing a standing wave pattern in the ring. Under these
circumstances, it is well known that a resonance condition exists, whose frequency is well approximated by:

\[ f_n = \frac{nc}{2L\sqrt{\varepsilon_{\text{eff}}(f)}} \]  

where \( n = 1, 2, \ldots \), is the mode number, \( c \) is the speed of light in vacuum, and \( \varepsilon_{\text{eff}}(f) \) is the frequency-dependent relative dielectric constant of the substrate supporting the ring.

\section*{B. MZI Ring Tuning}

To study the intrinsic tuning properties of the tunable ring, the structure shown in Figure 2 is considered.

It may be construed as consisting of an asymmetric ring made up of the parallel connection of one microstrip transmission line (TL) of length \( L_4 \) and width \( w \), and another TL consisting of the tandem connection of three TL segments of length and widths \( (L_1,w) \), \( (L_2,w_2) \), and \( (L_3,w) \), respectively. Other asymmetric MZI ring parameters are its mean radius, \( r \), and the mean circumference \( 2L = 2\pi r = L_1 + L_2 + L_3 + L_4 \), with:

\[ L_1 = r\left(\pi - \frac{\theta}{2} + \phi\right) \]  
\[ L_2 = r\theta \]  
\[ L_3 = r\left(\phi - \frac{\theta}{2}\right) \]  
\[ L_4 = r\pi \]  

For a given mean ring circumference, \( 2L \), tuning is effected by changing \( (L_2,w_2) \). This segment is positioned at the azimuthal angle \( \phi \) and has an angular extent \( \theta \), Figure 2.

In addition, at a frequency \( f \), the electrical parameters of the ring are given by the propagation constant of each TL segment \( j \), \( \gamma_j = z_j + j\beta_j = \alpha_j + j\beta_j \), where \( z_j \) and \( \beta_j \) are the attenuation and phase constants in segment \( j \), with \( \beta_j = 2\pi/\sqrt{\varepsilon_0\mu_0\varepsilon_{\text{eff}}(f)} = \beta_j \) and \( \mu_0 \) and \( \varepsilon_0 \) are the vacuum permeability and permittivity, respectively. The intrinsic tuning properties of the asymmetric ring are obtained by conceptually assuming it to be a two-port network, driven at its input and output ports by voltages and currents \( v_1, i_1 \) and \( v_2, i_2 \), respectively, Figure 2. The ring’s return loss, \( S_{R,11} = S_{R,11,N}/S_{R,D} \), and transmission, \( S_{R,21} = S_{R,21,N}/S_{R,D} \), were obtained analytically in terms of the overall ring admittance parameters \( (Y_{R,ij}) \), by computing the ABCD parameters of the two paths, converting the parameters of each path to admittance parameters, and adding the resulting Y-parameters of each path. The S-parameter expressions are long, so their numerators, \( S_{R,11}, S_{R,21} \), and their common denominator, \( S_{R,D} \), are given separately in Eq. (7a), where \( Z_0 \) is the reference impedance and the attenuation \( (\alpha) \) has been taken as zero. The S-parameters, computed with Eq. (7a), are plotted in Figure 3a; the transmission follows the same behavior of the symmetric ring [Eq. (1)]. There are no closed-form expressions for the resonance frequencies of the asymmetric MZI ring, since they derive from the transcendental equation obtained from seeking the roots of Eq. (7c) [shown as “from Eq. 7c” in Fig. 3c]; however, in a design, Eq. (2) is a good starting point.

The formulas are verified by comparing their predicted tuning frequencies, given by the minimum of \( S_{R,11} \) as computed from Eq. (7a), with the frequency of the minima obtained from CST [7] simulations, Figure 3b; the “kink” at \( \theta = 14^\circ \) is predicted by Eq. (7a) and is due to the increased proximity to the main resonance of a higher-frequency resonance as \( \theta \) approaches \( 14^\circ \). As clearly seen in Figure 3c, increasing the tuning sector length, \( \theta \), causes the resonance frequency to decrease, as should correspond to adding the tuning sector capacitance. It must be mentioned also that, given the nonlinear dependence of Eq. (7a) on \( \phi \) (through \( L_1 \) and \( L_3 \)), the tuning range is a function of the azimuthal angle \( \phi \).

\section*{C. MZI Ring Coupling to Transmission Line}

While the properties of ring-TL coupling have been studied [3,4], studies on the effect that ring tuning, as revealed

\[ L_1, Z_1 \], \( L_2, Z_2 \), and \( L_3, Z_3 \), of the ring tuned, are drawn in Figure 3a, with the frequency of the minima obtained from CST [7] simulations, Figure 3b; the “kink” at \( \theta = 14^\circ \) is predicted by Eq. (7a) and is due to the increased proximity to the main resonance of a higher-frequency resonance as \( \theta \) approaches \( 14^\circ \). As clearly seen in Figure 3c, increasing the tuning sector length, \( \theta \), causes the resonance frequency to decrease, as should correspond to adding the tuning sector capacitance. It must be mentioned also that, given the nonlinear dependence of Eq. (7a) on \( \phi \) (through \( L_1 \) and \( L_3 \)), the tuning range is a function of the azimuthal angle \( \phi \).
III. MZI-BASED DIELECTRIC RESONATOR TUNING CONCEPT AND EXPERIMENTAL VERIFICATION

A. Fundamental Concept

The idea of applying the tuned MZI ring to effect DR tuning derives from extending the DR-microstrip model [8], as shown next, may be exploited to tune a DR.

\[ S_{R1_{1,\lambda}} = 0.5 \csc [\ell_1 n_1] \left( -8 Z_0^2 Z_{01} Z_{02} + 2 Z_{01} Z_{02} \left( Z_{01} \cos f (L_1 + L_3 - L_4) n_1 + (4 Z_0^2 - Z_{01}^2) \cos f (L_1 + L_3 + L_4) n_1 \right) \cos [\ell_2 n_1] \right) \]

\[ - \left( Z_{01} \left( -2i Z_0 (Z_{01} - Z_{02}) (Z_{01} + Z_{02}) \cos f (L_1 - L_3 - L_4) n_1 + 2i Z_0 (Z_{01} - Z_{02}) (Z_{01} + Z_{02}) \cos f (L_1 - L_3 + L_4) n_1 \right) + Z_{01} (Z_{01}^2 + Z_{02}^2) \sin f (L_1 - L_3 - L_4) n_1 + Z_{01} (Z_{01}^2 + Z_{02}^2) \sin f (L_1 + L_3 - L_4) n_1 + Z_{01} (Z_{01} - Z_{02}) (Z_{01} + Z_{02}) \sin f (L_1 + L_3 - L_4) n_1 \right) \]

The study of MZI technique, shown in Figure 5, then it is clear that, in the immediate vicinity of the coupling region, over a certain extent of microstrip of length 2L, the MZI ring may be approximated by a microstrip of characteristic impedance \( Z_0 \) and propagation constant \( k \). In this context, this structure may also be modeled in a similar fashion to the microstrip line and, as shown next, may be exploited to tune a DR.

\[ S_{R2_{1,\lambda}} = -4i Z_0 Z_{01} \left( Z_{01} Z_{02} \csc [\ell_1 n_1] \left( Z_{01} Z_{02} \cos [\ell_2 n_1] \sin f (L_1 + L_3) n_1 + (Z_{01}^2 \cos [\ell_1 n_1] \cos [\ell_3 n_1] - Z_{01}^2 \sin [\ell_1 n_1] \sin [\ell_3 n_1] \sin [\ell_2 n_1] \right) \right) \right) \]

\[ S_{R0} = Z_{01} Z_{02} \csc [\ell_1 n_1] \left( 4 i Z_0 \cos [\ell_2 n_1] \left( Z_{01} \cos f (L_1 + L_3 - L_4) n_1 - (4 Z_0^2 + Z_{01}^2) \cos f (L_1 + L_3 + L_4) n_1 - 4Z_0 Z_{01} \sin f (L_1 + L_3 + L_4) n_1 \right) \right) \]

In particular, if we consider the MZI ring-DR structure, shown in Figure 5, then it is clear that, in the immediate vicinity of the coupling region, over a certain extent of microstrip of length 2L, the MZI ring may be approximated by a microstrip of characteristic impedance \( Z_0 \) and propagation constant \( k \). In this context, this structure may also be modeled in a similar fashion to the DR-microstrip of Figure 4, except that, due to the tuning produced by the loaded sector, see Figure 2, \( k \) becomes variable and so does \( N_t \). One concludes that a structure capable of tuning the frequency properties of a DR may be realized by configuring an MZI ring-DR-microstrip structure [9].

B. Basic MZI Ring-DR-Microstrip Structure

The basic MZI ring-DR-microstrip structure is depicted in Figure 6a. In it, the input microstrip power couples via the magnetic field (even mode) to the DR, Figure 6b, and from the DR to the MZI ring, where a standing wave is established, Figures 6b and 6c.

While the DR size is dictated by the frequency of interest, \( f_0 \), and may be determined from available software, e.g., from Transtech, Inc. [10], the selection of the MZI ring size is nontrivial [5]. On the one hand, since, as discussed by Lu and Ferendeci [3,4], coupling to the ring occurs via both odd (electric field coupling) and even (magnetic field coupling) modes, care must be exercised in choosing its size such that the resonance frequency for one of the ring’s even modes coincides with that of the DRs (i.e., \( f_0 \)). On the other hand, obtaining maximum overall quality factor requires minimizing the MZI ring’s radiated power [5]. Through studies of the ring’s radiation efficiency, under the conditions that it both exhibited an even mode coupling and reduced radiation efficiency at \( f_0 \), we determined that the MZI ring circumference should be chosen so the frequency of its second resonance mode (which is an even mode) coincided with \( f_0 \). For DR operation at \( f_0 \approx 10.2 \) GHz, we determined that an MZI ring with mean radius of 7.45 mm was adequate [5].

C. Tunable MZI Ring-DR-Microstrip Structure

Tuning of the DR, as discussed in Section II.B, is effected by varying the angular extent, \( \theta \), of a sector centered at...
an azimuth angle $\phi$, on the ring, Figure 7. As surmised from CST simulations of the structure, Figure 7, changes in sector length cause the magnetic field standing wave to rotate.

The effect of this rotation is to reduce the magnetic field that couples the MZI ring and DR, see Figure 8b.

Figure 4 (a) Distributed DR-microstrip model; (b) lumped DR-microstrip circuit model. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com]

The resulting DR tuning, manifested in the microstrip transmission ($S_{21}$), Figure 8c, is due to the concomitant change in $N_t$ and is explained by the MZI ring-DR-
microstrip circuit model, Figure 9. To extract this model, we began with the circuit model for the DR-microstrip, obtained with software by Transtech, Inc. [9].

Then, these initial RLC-transformer-microstrip circuit parameters were optimized so its transmission \( S_{21} \) would match the structures’ CST-computed \( S_{21} \), particularly, in the neighborhood of the resonance frequency. Next, the RLC elements for the MZI ring were computed from analytical formulas [1,2,11–13]. Fixing these set of parameters, the overall MZI ring-DR-microstrip \( S_{21} \), computed with CST [7] at various frequencies, were matched at each frequency in ADS [14] with the model in Figure 9a by using the turns ratio \( N_t \) as the single adjustable parameter. Comparisons of the circuit model results computed in ADS and the full-wave results computed in CST are shown in Figures 9b and 9c, respectively. As can be seen, the circuit model fits very well the full-wave CST simulations. Therefore, tuning is caused by variation in tuning sector length, which causes rotation of the magnetic field standing wave on the ring, which causes the magnitude of the field between the DR and MZI ring to change, which causes \( N_t \) to change and, consequently, causes the MZI ring’s impedance load “seen” by the DR to change, ultimately tuning it.

D. Deformed MZI Ring and Experimental Verification

As discussed above, the DR resonance frequency can be tuned by varying the sector length, \( \theta \), of the MZI ring coupled to it. However, in addition to frequency tuning range, our interest included elucidating how to obtain a concomitantly large \( Q \) [5]. Our investigations toward that goal concluded that the circular MZI ring is capable of effecting DR tuning while maintaining a relatively high \( Q \), but it exhibits a trade-off between them [5]. For example, for a target \( f_0 \sim 10 \) GHz and a nominal DR \( Q \) of 5000, we found that a tuning sector centered at \( \phi = 30^\circ \), exhibited a maximum \( Q \) of 663 and a tuning range of 58 MHz, but for \( \phi = 90^\circ \), the maximum \( Q \) was 923 and the tuning range was 20 MHz. The pursuit of improving this \( Q \)-tuning range trade-off, detailed in [5], led us to conceive of, and adopt, a configuration that would increase the MZI ring-DR-microstrip coupling, namely, the deformed ring structure [5], Figures 10a and 10b.

Theoretically, for the \( \phi = 30^\circ \) case (the \( \phi = 90^\circ \) case has no counterpart in the deformed ring structure), the deformed ring structure resulted in a maximum \( Q \) of 956 and a tuning range of 50 MHz [5]. This \( Q \) represents a 44% increase over that exhibited by the corresponding circular-ring structure and a 16% decrease in tuning range.

Figure 6 (a) MZI ring-DR-microstrip structure; (b) magnetic field in the structure; (c) magnetic field under the MZI ring along its mean radius, computed with CST. The structure has the parameters given in the caption of Figure 3, and the DR has a nominal \( Q \) of 5000. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

Figure 7 (a) Tunable MZI ring-DR-microstrip structure; right: (b)–(f) magnetic field under MZI ring at various tuning sector lengths \( \theta \) The tuned-sector width is 1 mm. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

bandwidth, for the same change in tuning-sector dimensions, suggesting a more favorable trade-off between Q and bandwidth for the deformed-sector ring [5]. Both the circular ring and the deformed ring structures were fabricated and tested; however, due to its superiority [5], we present here the results of the deformed ring-based structure, Figures 10a–10c. In particular, the variation in tuning sector angles was realized by fabricating three separate structures, like Figure 10c, each with \( \theta_5, \theta_6, \text{or } 10^\circ \).

Defining the tuning sector this way in the study provided a controlled way (i.e., one that is reproducible by others) of varying its size, which otherwise could be done by, e.g., manually placing on the ring gold ribbons of varying sizes, as is customary practice when “tuning” microwave integrated (i.e., alumina-based, etc.) circuits. The measured and CST-simulated \( S_{21} \) and \( S_{11} \) for these structures, shown in Figures 10d and 10e, exhibit a tuning range of \( \sim 32 \text{ MHz} \) about \( f_0 \sim 9.8 \text{ GHz} \) and a Q factor between 983 and 1089, for sector angles \( \theta_5, 6^\circ, \text{or } 10^\circ \), thus verifying the DR tuning concept. Implemented this way, DR-based circuits may be “manually” fine-tuned to a precise frequency. On the other hand, we envision in future work implementing DR tunability without manual intervention, e.g., through voltage control, by exploiting microelectromechanical systems/nanoelectromechanical systems fabrication technology [9]. While there is a discrepancy

**Figure 8** (a) MZI ring-DR-microstrip structure with tunable sector at \( \phi = 45^\circ \) showing line of symmetry \( y \); (b) magnetic field along line of symmetry \( y \) at various sectorial lengths \( \theta \); (c) \( S_{21} \) of structure in (a) showing tuning at various sectorial length \( \theta \). [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

**Figure 9** (a) MZI ring-DR-microstrip circuit model; (b) Variation of turns ratio as a function of tuning frequency; (c) \( S_{21} \) plot comparing circuit model and CST simulation results. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]
between measured and CST-simulated center frequencies, probably due to the effects of the housing, the tuning behavior is very similar in both cases. This eliminates the possibility of tuning behavior due to reproducibility issues and does corroborate the verification of the concept.

IV. CONCLUSION
A theoretical study to develop an understanding of the modulation of a metallic microwave-scale MZI coupled to a DR to effect tuning of the latter’s resonance frequency has been presented including an experimental verification of the concept. We have shown that the tuning range is related to the spatial overlap/coupling between MZI-ring and DR magnetic fields, which is varied when a tuning sector on the MZI ring causes the latter’s magnetic field standing wave to rotate. The experimental verification showed a tuning range of 32 MHz about 9.8 GHz, together with a Q between 983 and 1089. In future work, the concept studied here will be implemented by exploiting microelectromechanical systems/nanoelectromechanical systems fabrication technology to create DR-based

Figure 10  (a) Sketch of MZI ring-DR-microstrip structure (in practice, the indicated tuning sector may be implemented manually with a gold ribbon, until the desired resonance frequency is obtained); (b) details of pertinent deformed ring dimensions; (c) picture of typical fabricated MZI ring-DR-microstrip structure. Three structures with \( \theta = 2^\circ, 6^\circ, 10^\circ \) were fabricated; (d), (e) measured and CST-simulated \( S_{21} \) and \( S_{11} \) of fabricated structure shown in (c). Parameters: \( \phi = 30^\circ \); ring trace width, 0.5 mm; substrate thickness and relative permittivity, 0.635 mm and 10.2, respectively; metallization thickness, 0.017 mm; tuned-sector width, 1 mm; both the DR-microstrip and DR-ring separation distances were 0.1 mm; nominal DR Q was 4545 at 10 GHz and 5000 at 9.6 GHz, its diameter was 5.842 mm, its height, 2.7432 mm, and its relative permittivity 35.94. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]
microwave circuits, including oscillators and filters, which are tunable without manual intervention, e.g., through voltage control [9].

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BIOGRAPHIES

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